EFFECTIVE RESULTS FOR POINTS ON CERTAIN SUBVARIETIES OF TORI

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(joint work with J.-H. Evertse, K. Győry and C. Pontreau)

Choose an algebraic closure $\overline{\mathbb{Q}}$ of $\mathbb{Q}$. Recall that the group of $\overline{\mathbb{Q}}$-rational points of the N-dimensional torus is

$$\mathbb{G}_m^N(\overline{\mathbb{Q}}) = (\overline{\mathbb{Q}}^*)^N = \{\mathbf{x} = (x_1, \ldots, x_N) : x_i \in \overline{\mathbb{Q}}^* \text{ for } i = 1, \ldots, N\}$$

with coordinatewise multiplication, i.e., if $\mathbf{x} = (x_1, \ldots, x_N)$, $\mathbf{y} = (y_1, \ldots, y_N)$ then $\mathbf{xy} = (x_1y_1, \ldots, x_Ny_N)$. Denote by $h(x)$ the absolute logarithmic Weil height of $x \in \overline{\mathbb{Q}}$. Define the height and degree of $\mathbf{x} = (x_1, \ldots, x_N) \in (\overline{\mathbb{Q}}^*)^N$ by $h(\mathbf{x}) := \sum_{i=1}^N h(x_i)$, and $[\mathbb{Q}(x_1, \ldots, x_N) : \mathbb{Q}]$, respectively. Let $\mathcal{X}$ be an algebraic subvariety of $(\overline{\mathbb{Q}}^*)^N$ (i.e., the set of common zeros in $(\overline{\mathbb{Q}}^*)^N$ of a set of polynomials in $\overline{\mathbb{Q}}[X_1, \ldots, X_N]$), and Γ a finitely generated subgroup of $(\overline{\mathbb{Q}}^*)^N$. We want to study the intersection of $\mathcal{X}$ with any of the sets

$$\overline{\Gamma} := \left\{\mathbf{x} \in (\overline{\mathbb{Q}}^*)^N : \exists m \in \mathbb{Z}_{>0} \text{ with } \mathbf{x}^m \in \Gamma\right\} \quad \text{(the division group of } \Gamma\text{)},$$

$$\overline{\Gamma}_\varepsilon := \left\{\mathbf{x} \in (\overline{\mathbb{Q}}^*)^N : \exists \mathbf{y}, \mathbf{z} \in (\overline{\mathbb{Q}}^*)^N \text{ with } \mathbf{x} = \mathbf{yz},\ \mathbf{y} \in \overline{\Gamma},\ h(\mathbf{z}) < \varepsilon\right\},$$

$$C(\overline{\Gamma}, \varepsilon) := \Big\{\mathbf{x} \in (\overline{\mathbb{Q}}^*)^N : \exists \mathbf{y}, \mathbf{z} \in (\overline{\mathbb{Q}}^*)^N \\ \text{with } \mathbf{x} = \mathbf{yz},\ \mathbf{y} \in \overline{\Gamma},\ h(\mathbf{z}) < \varepsilon(1 + h(\mathbf{y}))\Big\},$$

where $\varepsilon > 0$.

We derive effective results for certain special classes of varieties $\mathcal{X}$. The classes of varieties we consider are such that they allow an application of logarithmic forms estimates. More precisely, we consider varieties in $(\overline{\mathbb{Q}}^*)^N$ given by equations $f_1(\mathbf{x}) = 0, \ldots, f_m(\mathbf{x}) = 0$ where each polynomial f_i is a binomial or trinomial.